Yield management under the real options scheme for optimal decision making in hotels*

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Abstract

Yield management is the process of actively managing inventory to maximize revenues. In this paper we present a model devised to apply yield management techniques using real options to the problem of optimal decision making when assigning rooms to hotel customers. Two different methods are proposed to carry out the evaluation: numerical resolution with the PDEs and MonteCarlo simulation. The achieved results using both methods are similar demonstrating the robustness of the simulation in this field and the model lends itself to be a tool for helping the hotel manager in his operational decision of whether or not giving a room to a potential client.

Keywords: real options, optimal decision making, yield management.

1. Introduction.

A model for the application of yield management techniques to the management of hotel rooms is presented in this article. The growing competitivity in the accommodation market, provoked in many cases by the supply excess, shows that a correct management should be done in order to maintain profitability.

Yield management is the perishable inventories managing process to maximize these stocks profits. This concept has its origin in the airlines industry where, in the take off moment, empty places represent profit opportunities lost. This approach is not only limited to the airlines sector, but is applicable to service industries, and in the hotels sector a real time tariffication is allowed, adapting the rooms price to the existing demand at every moment and maximizing that way sale profits.

One of the main characteristics that makes the profitability of the hotel industry so dependent of its price policy is the impossibility to stock the hotel product. In addition, the difficulty to modify the supply to adapt it to the movements of demand, together with the possibility to forecast its activity through reservations and historical experience, indicates that yield management techniques should be very well suited to optimize hotel operations.

* This work stems from the participation of the authors in a research project funded by "CICYT", title "Sistema avanzado de ayuda a la toma de decisiones a la gestión hotelera".

The problem to be solved in this sector is how to assign the available rooms, depending on the current supply and demand, in a way that the profit of the hotel is maximized. Because of the intrinsic dynamic characteristics of the system, the techniques used in the past don't take into account all the complexities of the problem. This is why the real options theory leads to a better solution than the static scenarios of the linear programming or queuing techniques.

The critical decision to be made by a hotel is whether to accept or not a room reservation for a future date. Until the moment of occupancy of a room, it will be assumed that its price will fluctuate with both a deterministic and a random component, which implies that the acceptance of a reservation currently involves a trade-off between the present cash flow and a possible higher cash flow in the future: should the room be rented today or should we wait for a better opportunity?. The exercise decision can occur at any time and every decision affects subsequent ones, since any room that gets reserved decreases the number of rooms which may be rented in the future.

A simple model for the hotel reservation process is described in this article. The problem is approached in two different ways: on the one hand by solving the partial differential equations that govern the process and on the other hand by using simulation techniques. The Least Squares MonteCarlo method will be used as well as the variance reduction techniques to minimize the computing time. The results obtained through both ways are presented and compared, showing the robustness and adequacy of the simulation in this field.

2. Model.

The model presented is based on the real options theory and looks forward to implement a useful decision making tool for the reservations of a hotel. The business considered has the following characteristics:

- M initially available rooms, all rooms are the same. In case that a hotel with different room categories is studied the problem could be broken down in k independent cases, each one with a unique type of room.

- Only one type of client and therefore demand. This demand will be directly related to the price evolution.

- Room's payment will be made upon reservation.

T is defined as reservation acceptance period. Once the time is over, the non-reserved rooms are lost revenue opportunities, because the rooms will be occupied in $t=T$. This reservation period will be divided in N intervals all of the same length ΔT. This means that we will face N times the problem of taking the decision due to the option being exercised on the discretized points. These time intervals are given by $0< t_1 < t_2 < \ldots < t_N = T$ where $t_j = j \cdot \Delta T$.

2.1. Demand model elaboration.

We consider the maximum number of rooms that can be reserved in an interval (demand in that period) as a linear function of the price. If the price of a room in a j period is called P_j, we could establish the demand expression as follows (1).

$$\text{Demand}\,(P_j) = \alpha + \beta \cdot P_j \qquad (1)$$

α, β coefficients could be constant in order to be able to program a simple example, or indexing them with time because of the modelizable demand difficulty not only depends on price but in factors such as the period of time considered or established prize of the competitors.

2.2. Price model.

The price will be represented by a random process. This could be justified because a hotel fixes his rates mainly looking at the competition and not in a randomised way. What is more the reservations done affect the availability and so the prices, conceived as a supply-offer relation that will vary with time. It is assumed that the price could be defined with a stochastic differential equation.

$$dP = H(P, t)\, dt + \sigma P\, dX \tag{2}$$

Where $P=P(t)$ is the room price in the t reservation moment. Constant σ volatility is considered to represent the rates constant relative fluctuation. Changing this constant by a time dependant function does not make any difference on the problem formulation. We choose in (2):

$$H(P, t) = \gamma\,(L(t) - P) \tag{3}$$

We consider that the price follows a mean reverting process with a rate γ and a time dependant mean prize L (t). This decision is made based on the sector study and means a good approximation due to the daily price variation being limited from the top and bottom. The lower bound exists because prices can not stay significantly below the marginal cost for too long or the hotel would have to exit the market. The top level is determined by the market competition strength and the clients supply. A growing trend for the mean price evolution will be assumed in a way that the nearer the end of the period is the greater is *L (t)*.

2.3. Problem approach.

A hotel with *T* reservation acceptance period is considered. In t=0, *M* available rooms exist for clients. In each *j* moment rooms could be rent with a P_j price that follows the stochastic process shown in (2). In each time step *N* in which could be make a decision the problem of how many room reservations is correct to accept arises.

A graph that shows the reservation situation in each time step (Figure 1) could be done and it could be interpret as a dynamic matrix of the hotel situation. Each node represents a hotel status: at time t_j, having *m* rooms reserved (*m* varying between 0 and *M*). The developed algorithm objective is to obtain the broken red line of the figure, that represents temporal succession of the hotel states that arrives to the profit maximum by the comparison of the cash flows expected facing with the decision to accept a reservation or wait for potential future (higher) revenues.

Considering the interval $t_j < t \leq t_{j+1}$, knowing the fact that the reservations could only be done in t_{j+1} (what implies no reservations at t=0). It is called E_m^j to the expected value of cash flows in case those m reservations exist on the step *j*. If a room is rented in the following instant, t_{j+1}, the cash flows expected value will be as follows:

$$\Psi = E_{m+1}^{j+1} + P_{j+1} \tag{4}$$

It is quickly deduced that the new value E_{m+1}^{j+1} is lower than E_m^j because now there is one less room to rent.

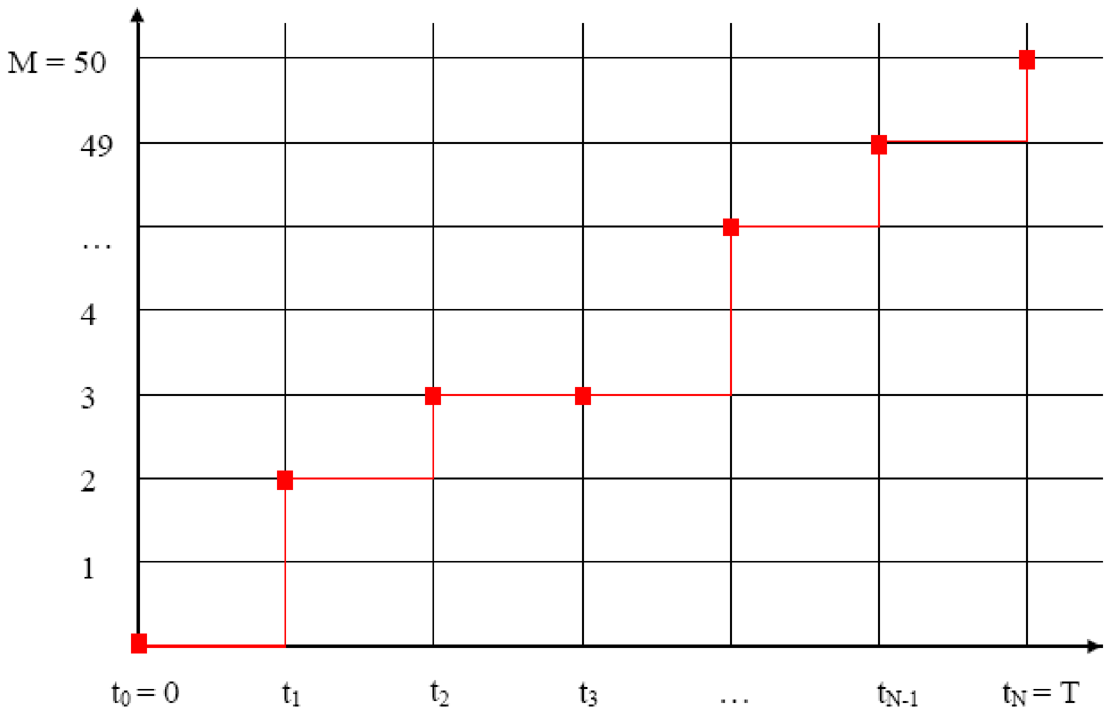

Figure 1. Reservation matrix.

In the same way if in the t_{j+1} instant k rooms are reserved the expected value is expressed by:

$$\Psi = E_{m+1}^{j+1} + k \cdot P_{j+1} \tag{5}$$

with $m+k \leq M$ because it can not be rent more rooms which are available. What is more, as the price is represented as a stochastic differential equation the cash value E_m^j could be considered as an European option with start in $t = t_j$ and end in $t = t_{j+1}$, and which payout could be represented by Ψ (5). The fact that it is European is because it could only be exercise in $t = t_{j+1}$.

With the purpose of maximizing revenues the decision criteria for the reservations acceptance will be the one which maximizes the total payout:

$$\Psi = Max\,(E_m^{j+1}\ ,\ E_{m+1}^{j+1} + P_{j+1}\ , \ldots ,\ E_{m+1}^{j+1} + k \cdot P_{j+1}) \tag{6}$$

where $m+k \leq M$ and $k \leq Demand\,(P_{j+1})$.

This method is applied for each one of the steps which reservations period is composed. In the last interval ($j=N$), the payout is the maximum number of rooms that remain available taking into account the constraint due to the demand. It is because an empty room at $t=T$ represents a lost revenue opportunity.

2.4. Partial Derivative Equation Resolution (PDE method).

The objective is to determine the option value of rent the room in *t* instant, knowing that if the reservation in *T* is not done the opportunity to rent the room is lost. The value of this option is *E (P, t;T)*. The hypothesis that the solution is a risk-neutral measure is done and knowing the Bellman principle the next partial derivative equation is obtained:

$$\frac{\partial E}{\partial t} + \frac{1}{2}\sigma^2 P^2 \frac{\partial^2 E}{\partial^2 P} + \gamma(L - P)\frac{\partial E}{\partial P} - rE = 0 \quad (7)$$

where r is the risk-free rate discount. For more comfort (7) is transformed substituting the time variable by a τ local variable defined upon the rectangle (for each level of occupation) with extremes on t_{j+1} . and t_j. So, the partial derivative equation that rules the process is:

$$\frac{\partial E}{\partial \tau} = \frac{1}{2}\sigma^2 P^2 \frac{\partial^2 E}{\partial^2 P} + \alpha(L - P)\frac{\partial E}{\partial P} - rE \quad (8)$$

The initial condition for this equation is obtained from (6) and is:

$$\Psi = Max\ (E^m_{j+1}\ ,\ E^{m+1}_{j+1} + P_{j+1}\ ,\ \ldots\ ,\ E^{m+k}_{j+1} + k\cdot P_{j+1}) \quad (9)$$

with $m+k \leq M$ and $k \leq$ *Demand* (P_{j+1}). It is necessary to define the boundary conditions. It is demonstrated that for *P*=0 is not necessary a boundary condition because $\gamma \cdot L > 0$. However for *P* large values, is evident that *E* will be linearly dependant of P. For the last time interval we can write down an exact solution of the form:

$$E^N_m(\tau) = A(\tau) + B(\tau)P \quad (10)$$

due to the initial condition E^N_m (0) =K·P, where K is a fixed constant.

From here a numerical resolution for solving the partial derivative equations based on the finite differences method was developed, this procedure yields to a tridiagonal equations system solved by the programmed calculus model.

2.5. Simulating solution.

MonteCarlo method is a statistical approximation to the problem. *N* different price paths are generated following the model expressed by (2) along the reservations acceptance period. To solve the problem is necessary to go to the ending instant of time *(t=T)* in which *m* reservation rooms are considered. For each path of price *i* we calculate the maximum number of rooms which could be reserved in instant *T*:

$$K^i_T = Min(M - m\,, Demand(P^i_T)) \quad (11)$$

where P^i_T is the price in *T* for the *i* path. This procedure is done for each path obtaining the decision vector:

$$D_{m,T} = \begin{pmatrix} K_T^1 \\ K_T^2 \\ \dots \\ \dots \\ K_T^n \end{pmatrix} \tag{12}$$

Coming back to the previous instant it is calculated again the maximum number of rooms for each path that could be reserved considering that *m* were already rent in past instants. The received payout in *T-1* will be:

$$\Psi = Max\ (E_{T-1}^{m}\ ,\ E_{T-1}^{m+1}\ +\ P_{T-1}^{i}\ ,\ \dots\ ,\ E_{T-1}^{m+k}\ +\ K_{T-1}^{i}\cdot P_{T-1}^{i}\ ) \tag{13}$$

And in a generic step time j the layout can be expressed as follows:

$$\Psi = Max\ (E_{j+1}^{m}\ ,\ E_{j+1}^{m+1}\ +\ P_{j+1}^{i}\ ,\ \dots\ ,\ E_{j+1}^{m+k}\ +\ K_{j+1}^{i}\cdot P_{j+1}^{i}\ ) \tag{14}$$

At each instant of time a regression is done by the LSM method. That way the expected cash flows payout could be calculated for each price path obtaining a decision vector that represents the number of rooms that should be reserved in each instant of time for each *i* path, knowing that *m* rooms have been already reserved. That way is proceed until the arrival to the initial instant, obtaining a matrix formed by the decisions made in each instant for each path and the cash flow expected values.

3. Results.

The outputs of the model can be summarized as follows:

- Present value of the earnings associated to an optimal room management. Here, results depart heavily from the standard NPV rule results, mainly due to the value of the real options involved in the day to day management of the hotel: basically, the option to wait and (potentially) hire rooms at a higher price or for a longer period of time.

- Practical management decision tool. The model confronts the immediate earnings associated to rent the room now (exercise value) against the expected earnings associated to future inflows. Given the historical data and the demand/price function, the hotel manager can actually decide which strategy gives a higher yield.

The results obtained by both methods for a certain numeric example are presented below. A 3 month reservation acceptance period divided in 12 steps in a little facility with 50 rooms has been considered. The free risk rate is 5% and the mean reverting parameter (γ) is 180. In what concerns the demand the parameters of equation (1) have the values α=20 and β=-0.07 respectively.

3.1. Partial derivative equation resolution.

For the indicated values the expected cash flow value at the initial interval is *E*=1467.67 €. Let us focus on the first decision moment behaviour shown in Figure 2, where the number of accepted reservations is represented against price.

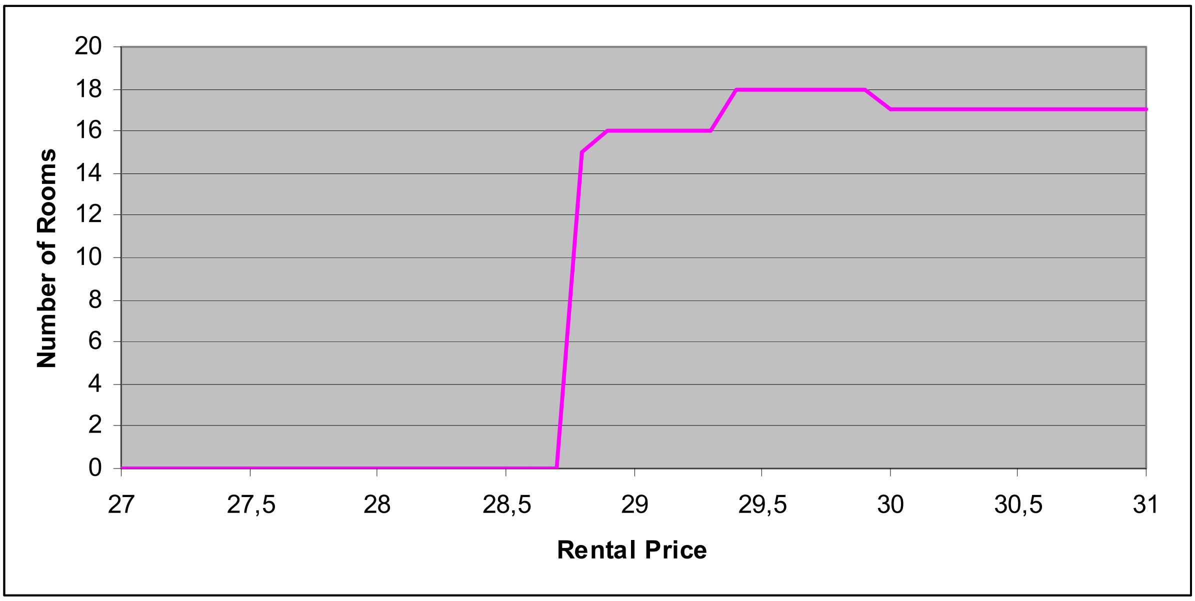

Figure 2. Price and availability at initial booking. PDEs.

Reservations are accepted from 28.8€. All the possible rooms for that demand are booked for prices between 29.4€ and 30€, and for prices over 30€ the demand decreases.

This behaviour seems to be reasonable since for low prices there is no advantage at the first instant for the hotel manager to accept reservations. Due to the fact that the mean reverting parameter is high (γ=180) a rising evolution of the price during the booking period is granted at a low risk. Therefore, if the initial price is low the best decision is to wait as the graphic shows.

Simulation resolution.

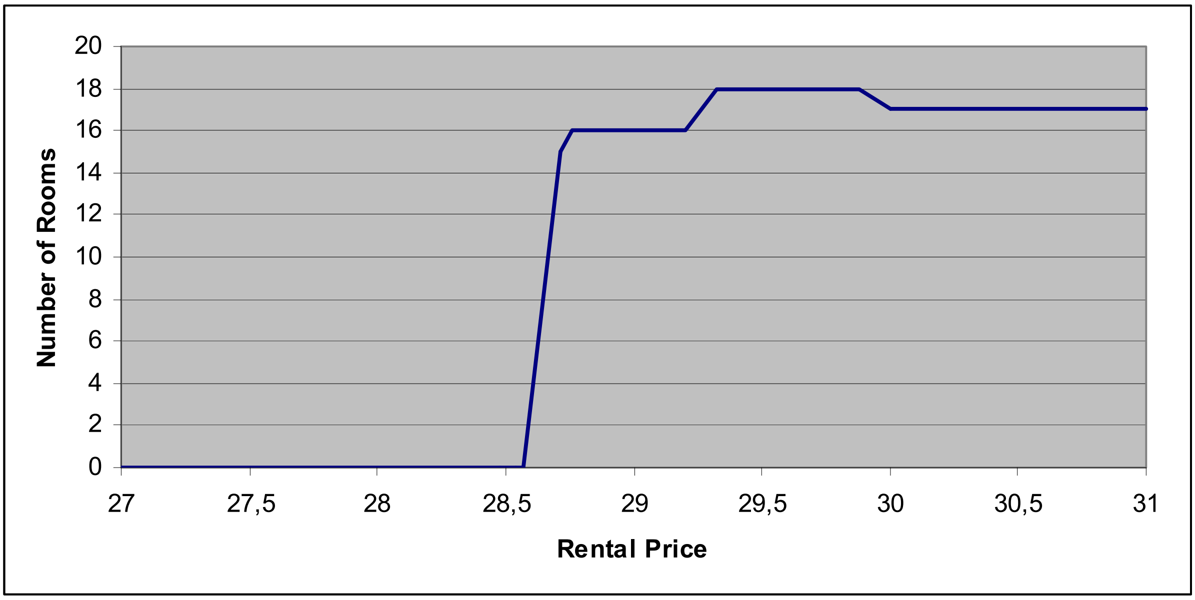

Figure 3. Price and availability at initial booking. MonteCarlo Simulation.

In this case the expected cash flow value at the initial time for the numerical example is *E*=1455.83€. The results have been obtained with 10,000 simulations. Analysing the behaviour at the first decision moment (Figure 3) with this method, it is observed that for prices below 28.7€ no booking is accepted. Sixteen rooms are rented for prices between 28.7€ and 29.2€, reaching a peak of 18 reservations. The demand decreases for prices over 30€.

A sensitivity analysis for the mean reverting parameter has been done and it is observed that the lower the parameter γ is the sooner the rooms are rented, so the reservations are accepted for lower prices. The reason is, as formerly commented, that as the value of γ decreases the growing price evolution uncertainty increases.

3.2. Results comparison.

The results obtained with the different methods presented are very similar. For example, when the expected value of the hotel business is calculated, the results reached with both methods differ in less than 0.8%. In addition, the number of booked rooms in relation to their price graphics is almost identical. If we study in more detail the values obtained for γ=180, we can observe the similarity of results (Table 1).

Table 1. Comparison between both methods.

Number of Rooms	**Rental Price**	
	PDEs	MonteCarlo Simulation
0	28.5	28.478
	28.6	28.542
	28.7	28.566
15	28.8	28.717
16	28.9	28.762
	29	28.764
	29.1	28.888
	29.2	28.931
	29.3	28.934
		29.047
		29.054
		29.060
		29.123
		29.143
		29.166
		29.181
		29.199
18	29.4	29.326
		29.370

With the first method we could start booking rooms from a price of 28.8€, according to the other one for 28.717€, a really similar result taking into account that we are using iterative methods that tend to the exact solution. The transition between sixteen and eighteen rooms occurs for a price of 29.4€ and 29.32€ respectively.

4. Conclusions.

The objective of the research presented in this paper was to develop a tool that would allow to improve the hotel management by means of using the real options theory. More specifically it was looking forward to being able to apply MonteCarlo simulation techniques, which have proved to be very useful when applied in other fields and which have been used for the first time to solve a problem of yield management in the hotel sector.

The resolution of the problem using the real options theory offers robust results. Two different methods have been proposed to carry out the evaluation: numerical resolution with the PDEs that model the process and MonteCarlo simulation method approximation. The achieved results using both methods are similar and thus open the door to the MonteCarlo Simulation resolution methods for the decision making on the hotel sector management.

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