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Tri–objective Stochastic Model for Designing Multi-layer Reverse Logistic Systems[1]

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1. Introduction

Reverse Logistics is a topic which has drawn ever more attention over the last decades. Society is more and more concerned with environmental aspects. Research has also followed this trend and has addressed several problems related to Reverse Logistics.

One of these problems is deciding where to locate facilities to treat all waste generated in a particular region. In order for waste to be appropriately handled, waste has to be collected and treated, which it either means disposing of it or obtaining sellable products. Generally, there are multiple types of facilities to locate and decisions to be made.

Besides, several criteria can be considered. Not only is cost a major concern for designing reverse networks, but other issues can be also important such as carbon dioxide emissions or the obnoxious effect derived from these facilities.

2. Literature review

The research on reverse logistics planning was triggered by a some pioneering works (Barros et al 1998; Fleischmann et al 1997; Jayaraman et al 1999) and has experienced a strong development over the last decade. The strong development of supply chain planning has also led to more attention being paid to reverse activities that must follow the typical forward activities defined by supply chain management.

Ever since, there have been different papers addressing where to locate facilities in a Reverse Logistic system. Each of them present different characteristics which correspond to different decisions to be taken.

As will be described, the model in this paper is a comprehensive model which includes in a single model most of those decisions so far addressed separately.

3. Problem description

3.1. System operation

Let us assume that a company is in charge of managing the whole process within a region: from waste collection to its treatment in specialized facilities. There exist different types of waste products generated in towns across the whole region.

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Waste products can be sent to collection centres, where they are compressed and sent to a recovery centres. Alternatively, waste products can be sent directly to treatment plants.

Collection centres can compact a particular waste product if it has been properly prepared. Collection centres' cost can be compensated by the reduction costs obtaining when transporting compacted waste.

Waste products are treated in recovery centres. A recovery centre can process a waste product if the necessary technology has been installed. For every waste product different technologies can be chosen (such as manual, automatic, semi-automatic…)

Once a type of technology has been chosen, its capacity is to be determined. This is done in modular terms, which means that for a particular technology and a particular waste product, one, two, etc. modules can be installed.

After treated, waste products are turned into final products, which can be sold in the market obtaining an income. Besides, an amount of residual product is generated, that can be either incinerated or sent to a landfill. This last case can be treated as a final product which gives certain revenue.

Each type of technology has a maximum transformation rates from waste products into final, which limits the amounts of final products to be produced.

These amounts are also limited because there exist upper bounds for the amount of final products that can be sold in the market and because there exist lower bounds for the amounts of final products that should be obtained.

As to the criteria to assess solutions, three major concerns are addressed: the total cost, the obnoxious effect and the CO2 emissions.

3.2. Uncertainty

As in long term decisions (especially for non-mature activities) there are uncertain date. In this case, two sources:

- transportation costs, for which three different sets of values have been considered, low, medium and high costs); and
- waste generation, for which three different sets of values have also been considered (low, medium and high generation).

As a result, when combining those sets, nine scenarios are considered.

3.3. Decisions involved

For this problem, the decisions to be made are the following ones (for which the corresponding decision variables will be defined later).

- Where to locate collection centres among a set of potential locations, and what type of waste products can processed at each of them.
- Where to locate recovery centres among another set of locations, what waste products can be treated.
- Where to locate the incinerator if anywhere.
- What technology is to be installed and with which capacity in every recovery centre.
- What amounts of waste product are to be sent from each origin to each collection centre and from each origin to each recovery centre.

- What amounts of final products to be produced and sold, what amounts of residual products should be dumped into the landfill, and what should be incinerated.

3.4. Bounty criteria

In order to assess different configurations for the network, three different criteria have been considered: costs, obnoxious effects and CO_2 emissions, which depend on the following aspects.

3.4.1. Costs

- Collection costs, for transporting (non compacted) waste products from origins to collection centres or to recovery centres.
- Transfer costs, for transporting (compacted) waste products from collection centres to recovery centres.
- Set-up costs for every type of facility and set-up costs for enabling a particular facility to treat a specific waste product.
- Benefits derived from selling final products in the market.
- Disposal tax, paid for disposing of residual product which is sent to the landfill.

3.4.2. Obnoxious effect

In principle, facilities are not convenient for the populations nearby. Every facility has a particular obnoxious effect which depends on the distance from that facility to the town that suffers the obnoxious effect.

On the other hand, the provide employment and frequently are accompanied with tax reductions. This should be considered.

In all, the expressions for the obnoxious effects are the following ones:

3.4.3. CO_2 emisions

The main sources for CO_2 emissions are transportation of material among different locations, facility operations and power supply consumptions and savings.

4. Formulation

4.1. Sets

The following sets need to be defined. $\boldsymbol{P_W}$: set of disposal types. $\boldsymbol{P_F}$: set of valuable commodities. $\boldsymbol{G}$: set of different technologies available for the recovery centres. $\boldsymbol{J}$: set of origins of the disposals. $\boldsymbol{I_C}$: set of potential locations for collection centres. $\boldsymbol{I_R}$: set of potential locations for recovery centres. $\boldsymbol{I_L}$: set of potential locations for landfills. $\boldsymbol{I_I}$: set of potential locations for the incinerator. $\boldsymbol{S}$: set of scenarios.

4.2. Parameters

General Parameters

$\boldsymbol{KC_p}$: capacity of a collection centre for processing product p in P_W. $\boldsymbol{Q_{pg}}$: capacity of a module of technology g in G when processing product p in P_W . $\boldsymbol{KR_{ipg}}$: maximum number of modules of technology g in G that can be installed at recovery centre i in I_C to process product p in P_W. $\boldsymbol{A_{pqg}}$: maximum proportion of valuable commodity p in P_F that can be obtained from product q in P_W when using technology g in G. $\boldsymbol{D^O_{ij}}$: road distance between origin j in J and facility i in $I_C \cup I_R$. $\mathbf{D^{NO}_{ii'}}$: road distance between facility i in I_R and facility i in $I_C \cup I_L$. $\boldsymbol{MIN_{pq}}$: the minimum rate imposed for the conversion of product q in P_W into product p in P_F. $\boldsymbol{MAX_p}$:the maximum amount of valuable commodity p in P_F that can be sold.

Costs/Profits parameters

$\boldsymbol{FC_i}$: fixed cost for installing a collection centre at i in I_C. $\boldsymbol{FR_i}$: fixed cost for installing a recovery centre at i in I_R. $\boldsymbol{FL_i}$: fixed cost for installing a landfill at i in I_L. $\boldsymbol{FCP_p}$: fixed cost for preparing a collection centre for receiving disposal p in P_W. $\boldsymbol{FRPG_{ipg}}$: fixed cost for installing a module of technology g in G in location i in I_R for processing disposal p in P_W. $\boldsymbol{B_p}$: unitary profit for valuable commodity p in P_F. **DT**: disposal tax per unit of residual product.

Uncertainty parameters

$\boldsymbol{O_{jps}}$: amount of disposal p in P_W originated at origin j in J in scenario s *in S*. $\boldsymbol{CTO_{ps}}$: cost (monetary units per unit and per km) for shipping one unit of product p in P_W from an origin to a collection centre or to a recovery centre. $\boldsymbol{CTC_{ps}}$: cost (monetary units per unit and per km) for shipping one unit of product p in P_W from a collection centre to a recovery centre. $\boldsymbol{CTR_s}$: cost (monetary units per unit and per km) for shipping one unit of residual product from a recovery centre to a landfill.

Obnoxious effects

The obnoxious effects are respectively $\boldsymbol{OE_{ip}^C}$, $\boldsymbol{OE_{ip}^R}$, $\boldsymbol{OE_i^L}$, $\boldsymbol{OE_i^I}$, which are the effects of locating in location j a collection centre for treating product p in P_W, a recovery centre, a landfill and an incinerator (respectively).

The obnoxious effects denoted above as are generally assumed to be functions of the Euclidean distances between origins (populations) and facilities. However, other factors are important for determining these effects namely, the populations involved and the type of disposal considered. Due to lack of space, further details cannot be provided.

Emissions parameters

$\boldsymbol{EN_p^C}$: energy consumption in collection centre i in I_C when processing p in P_W. $\boldsymbol{EN_{pg}^R}$: energy consumption in a recovery centre i in I_R when processing p in P_W with technology g in G. $\boldsymbol{EN^I}$: energy production in an incinerator in i in I_I. $\boldsymbol{E^I}$**:** CO_2 emitted by an incinerator i in I_I. $\boldsymbol{E^{AL}}$: CO_2 emitted by a ash landfill associated to the incinerator in i in I_I. $\boldsymbol{E^L}$: CO_2 emitted by a landfill located in i in I_L. $\boldsymbol{EEN}$: CO_2 emissions as a result of the energy consumption. $\boldsymbol{TE^O}$: CO_2 emitted by trucks collecting disposal from towns. $\boldsymbol{TE^{NO}}$: CO_2 emitted by trucks transporting waste among reverse facilities. $\boldsymbol{F^O}$: compactness factor of a collection truck.

4.3. Variables

Strategic Decision variables

$\boldsymbol{y_i^C}$ = 1 if a collection centre is installed at i in I_C, 0 otherwise. $\boldsymbol{y_i^R}$ = 1 if a recovery centre installed at i in I_R, 0 otherwise. $\boldsymbol{y_i^L}$ = 1 if a landfill is installed at i in I_L, 0 otherwise. $\boldsymbol{y_i^I}$ = 1 if an incinerator is installed at i in I_I, 0 otherwise. $\boldsymbol{w_{ip}^C}$ = 1if collection centre i in I_C processes disposal p in P_W, 0 otherwise. $\boldsymbol{w_{ip}^R}$ = 1if recovery centre i in I_R is processing disposal p in P_W, 0 otherwise. $\boldsymbol{z_{ipg}^R}$ = 1 if recovery centre i in I_R processes disposal p in P_W using technology g in G, 0 otherwise. $\boldsymbol{n_{ipg}^R}$ = number of modules of technology g in G installed in recovery centre i in I_R to treat disposal p in P_W.

Tactical/Operational Decision variables

$\boldsymbol{x_{ijps}^{OC}}$: amount of disposal p in P_W sent from origin j in J to collection centre i in I_C. $\boldsymbol{x_{ijps}^{OR}}$: amount of disposal p in P_W sent from client j in J to recovery centre i in I_R. $\boldsymbol{x_{ii'ps}^{OC}}$: amount of disposal p in P_W sent from collection centre i in I_C to recovery centre i' in I_C. $\boldsymbol{x_{iis}^{RL}}$: amount of residual disposal obtained in recovery centre *i* in I_R sent to landfill *i´* in I_L. **x_{ipqg}^{RV}**: amount of valuable commodity *p* in P_F obtained in recovery centre *i* in I_R from product q in P_W using technology *g* in *G*. **$x_{ii's}^{RI}$**: amount of residual disposal obtained in recovery centre *i* in I_R sent to incinatory *i´* in I_I .

4.4. Objective functions

3.4.4. Cost function

$$
\begin{aligned}
\text{MIN} \quad & \sum_{i\in\mathcal{I}_C} FC_i y_i^C + \sum_{i\in\mathcal{I}_R} FR_i y_i^R + \sum_{i\in\mathcal{I}_I} FL_i y_i^I + \sum_{i\in\mathcal{I}_L} FI_i y_i^L \\
& + \sum_{p\in\mathcal{P}_W} \left(FCP_p \sum_{i\in\mathcal{I}_C} w_{ip}^C \right) + \sum_{i\in\mathcal{I}_R} \sum_{p\in\mathcal{P}_W} \sum_{g\in\mathcal{G}} FRPG_{ipg} n_{ipg}^R \\
& + \sum_{s\in\mathcal{S}} \rho_s \Bigg[\sum_{j\in\mathcal{J}} \sum_{i\in\mathcal{I}_C} \sum_{p\in\mathcal{P}_W} CTO_{ps} D_{ij}^O x_{ijps}^{OC} + \sum_{j\in\mathcal{J}} \sum_{i\in\mathcal{I}_R} \sum_{p\in\mathcal{P}_W} CTO_{ps} D_{ij}^O x_{ijps}^{OR} \\
& \quad + \sum_{i\in\mathcal{I}_C} \sum_{i'\in\mathcal{I}_R} \sum_{p\in\mathcal{P}_W} CTC_{ps} D_{ii'}^{NO} x_{ii'ps}^{CR} + \sum_{i\in\mathcal{I}_R} \sum_{i'\in\mathcal{I}_I} CTRI \times D_{ii'}^{NO} x_{ii's}^{RI} \\
& \quad + \sum_{i\in\mathcal{I}_R} \sum_{i'\in\mathcal{I}_L} CTRL \times D_{ii'}^{NO} x_{ii's}^{RL} + DT \sum_{i\in\mathcal{I}_R} \sum_{i'\in\mathcal{I}_L} x_{ii's}^{RL} \\
& \quad - \sum_{i\in\mathcal{I}_R} \sum_{p\in\mathcal{P}_F} \sum_{q\in\mathcal{P}_W} B_p \sum_{g\in\mathcal{G}} x_{ipqgs}^{RV} + \sum_{i\in\mathcal{I}_R} \sum_{i'\in\mathcal{I}_I} EN^I (FIP - B^I) x_{ii's}^{RI} \Bigg]
\end{aligned} \tag{1}
$$

This function is composed of several terms. Those of the first line refer to the fixed costs for the installing facilities (collection centres, recovery centres, incinerators and, finally, landfills), whereas the second line refers to the fixed costs for enabling the treatment of a product in every collection centre and the cost for the number of modules for each technology and each product installed in each recovery centre. The last four lines is the expected value for the operative costs, which depend on the set of scenarios and their probabilities. For each scenario, the operative costs are those derived from material transportation among facilities (first five terms), those corresponding to the disposal taxes, the benefits obtained from selling products (negative cost) and, finally, the costs derived from the energy consumption (discounting the benefits from the energy obtained in the incinerator).

Obnoxious effect function

$$
\text{MIN} \quad \sum_{i\in\mathcal{I}_C} \sum_{p\in\mathcal{P}_W} OE_{ip}^C w_{ip}^C + \sum_{i\in\mathcal{I}_R} \sum_{p\in\mathcal{P}_W} OE_{ip}^R \sum_{g\in\mathcal{G}} z_{ipg}^R + \sum_{i\in\mathcal{I}_L} OE_i^L y_i^L \tag{2}
$$

The obnoxious effects are associated to the type of facilites and their location. There are as many terms in the objective function as types of facilities.

Emissions function

$$\begin{aligned}\text{MIN} \quad \sum_{s\in\mathcal{S}} \rho_s \Bigg[& \sum_{j\in\mathcal{J}} \sum_{i\in\mathcal{I}_C} \sum_{p\in\mathcal{P}_W} x_{ijps}^{OC} EN_p^C EEN + \sum_{j\in\mathcal{J}} \sum_{i\in\mathcal{I}_R} \sum_{p\in\mathcal{P}_W} \sum_{g\in\mathcal{G}} x_{ijps}^{OR} EN_{pg}^R EEN \\ & + \sum_{i\in\mathcal{I}_C} \sum_{i'\in\mathcal{I}_R} \sum_{p\in\mathcal{P}_W} \sum_{g\in\mathcal{G}} x_{ii'ps}^{CR} EN_{pg}^R EEN + \sum_{i\in\mathcal{I}_R} \sum_{i'\in\mathcal{I}_L} x_{ii's}^{RL} E^L \\ & + \sum_{i\in\mathcal{I}_R} \sum_{i'\in\mathcal{I}_I} \left(x_{ii's}^{RI} E^I - 0{,}8 x_{ii's}^{RI} EN^I EEN + \frac{x_{ii's}^{RI}}{3} E^{AL} \right) \\ & + \sum_{j\in\mathcal{J}} \sum_{i\in\mathcal{I}_C} \sum_{p\in\mathcal{P}_W} F_p^O x_{ijps}^{OC} D_{ij}^O + \sum_{j\in\mathcal{J}} \sum_{i\in\mathcal{I}_R} \sum_{p\in\mathcal{P}_W} F_p^O x_{ijps}^{OR} D_{ij}^O \\ & + \sum_{i\in\mathcal{I}_C} \sum_{i'\in\mathcal{I}_R} \sum_{p\in\mathcal{P}_W} F_p^{NO} x_{ii'ps}^{CR} D_{ij}^{NO} + \sum_{i\in\mathcal{I}_R} \sum_{i'\in\mathcal{I}_I} F^I x_{ii's}^{RI} D_{ij}^{NO} + \sum_{i\in\mathcal{I}_R} \sum_{i'\in\mathcal{I}_L} F^L x_{ii's}^{RL} D_{ij}^{NO} \Bigg] \end{aligned} \tag{3}$$

This last function computes the total amount of CO_2 produced by the system. The first four terms represent the CO2 emissions due to the amount of materials treated in each of them (collection centres, recovery centres and landfills). The third line corresponds to the emissions of the incinerator, where the energy produced is offset by the savings derived from the energy it produces and the emissions from the ash landfill is added. The last five terms compute the costs corresponding to waste transportation, from towns to collection centres, from towns to recovery centres, from collection to recovery centres, from recovery centres to incinerators and, finally, from recovery centres to landfills.

4.5. Constraints

$$\sum_{i\in\mathcal{I}_C} x_{ijps}^{OC} + \sum_{i\in\mathcal{I}_R} x_{ijps}^{OR} = O_{jps} \qquad p\in\mathcal{P}_W,\ j\in\mathcal{J},\ s\in\mathcal{S} \tag{4}$$

$$\sum_{j\in\mathcal{J}} x_{ijps}^{OC} = \sum_{i'\in\mathcal{I}_R} x_{ii'ps}^{CR} \qquad i\in\mathcal{I}_C,\ p\in\mathcal{P}_W,\ s\in\mathcal{S} \tag{5}$$

$$A_{pqg} \left(\sum_{j\in\mathcal{J}} x_{ijqs}^{OR} + \sum_{i'\in\mathcal{I}_C} x_{i'iqs}^{CR} \right) \geq x_{ipqgs}^{RV} \tag{6}$$

$$i\in\mathcal{I}_R,\ q\in\mathcal{P}_W,\ p\in\mathcal{P}_F,\ g\in\mathcal{G},\ s\in\mathcal{S}$$

$$x_{ipqgs}^{RV} \leq \left(\sum_{j\in\mathcal{J}} O_{jq} \right) z_{ipg}^R \tag{7}$$

$$i\in\mathcal{I}_R,\ q\in\mathcal{P}_W,\ p\in\mathcal{P}_F,\ g\in\mathcal{G},\ s\in\mathcal{S}$$

$$\sum_{i'\in\mathcal{I}_L} x_{ii's}^{RL} + \sum_{i'\in\mathcal{I}_I} x_{ii's}^{RI} = \sum_{q\in\mathcal{P}_W} \left(\sum_{j\in\mathcal{J}} x_{ijqs}^{OR} + \sum_{i'\in\mathcal{I}_C} x_{i'iqs}^{CR} \right) - \sum_{q\in\mathcal{P}_W} \sum_{p\in\mathcal{P}_F} \sum_{g\in\mathcal{G}} x_{ipqgs}^{RV} \tag{8}$$

$$i\in\mathcal{I}_R,\ s\in\mathcal{S}$$

$$\sum_{j\in\mathcal{J}} x^{OR}_{ijps} + \sum_{i'\in\mathcal{I}_C} x^{CR}_{i'ips} \le \sum_{g\in\mathcal{G}} Q_{pg} n^{R}_{ipg} \qquad i\in\mathcal{I}_R,\ p\in\mathcal{P}_W,\ s\in\mathcal{S} \tag{9}$$

$$\sum_{i\in\mathcal{I}_R}\sum_{g\in\mathcal{G}} x^{RV}_{ipqgs} \ge MIN_{pq}\left(\sum_{j\in\mathcal{J}} O_{jqs}\right) \qquad q\in\mathcal{P}_W,\ p\in\mathcal{P}_F,\ s\in\mathcal{S} \tag{10}$$

$$\sum_{i\in\mathcal{I}_R}\sum_{q\in\mathcal{P}_W}\sum_{g\in\mathcal{G}} x^{RV}_{ipqgs} \le MAX_p \qquad p\in\mathcal{P}_F,\ s\in\mathcal{S} \tag{11}$$

$$\omega^{C}_{ip} \le y^{C}_{i} \qquad i\in\mathcal{I}_C,\ p\in\mathcal{P}_W \tag{12}$$

$$\sum_{j\in\mathcal{J}} x^{OC}_{ijps} \le KC_p\,\omega^{C}_{ip} \qquad i\in\mathcal{I}_C,\ p\in\mathcal{P}_W,\ s\in\mathcal{S} \tag{13}$$

$$\sum_{g\in\mathcal{G}} z^{R}_{ipg} \le y^{R}_{i} \qquad i\in\mathcal{I}_R,\ p\in\mathcal{P}_W \tag{14}$$

$$n^{R}_{ipg} \le KR_{ipg}\, z^{R}_{ipg} \qquad i\in\mathcal{I}_R,\ p\in\mathcal{P}_W,\ g\in\mathcal{G} \tag{15}$$

$$x^{RI}_{ii's} \le \left(\sum_{j\in\mathcal{J}}\sum_{p\in\mathcal{P}_W} O_{jps}\right) y^{I}_{i} \qquad i\in\mathcal{I}_R,\ i'\in\mathcal{I}_I,\ s\in\mathcal{S} \tag{16}$$

$$\sum_{i\in\mathcal{I}_I} y^{I}_{i} \le 1 \qquad i\in\mathcal{I}_I \tag{17}$$

$$x^{RL}_{ii's} \le \left(\sum_{j\in\mathcal{J}}\sum_{p\in\mathcal{P}_W} O_{jps}\right) y^{L}_{i} \qquad i\in\mathcal{I}_R,\ i'\in\mathcal{I}_L,\ s\in\mathcal{S} \tag{18}$$

Constraints (4) assure that for each type of disposal and for each origin the amount generated in the origin is sent either to a collection or to a recovery centre. Constraints (5) are the flow conservation constraints for each collection centre and for each type of disposal. Constraints (6) restrict the amount of every valuable commodity obtained from every waste product in a recovery centre for every technology. It depends on the amount of received waste product and the conversion rates allowed by that technology. Constraints (7) assure that the amount of some valuable commodity obtained from some disposal in a recovery centre can only be greater than 0 if that recovery centre is in fact treating that disposal. Constraints (8) assure, for each recovery centre, that the amount of disposal not transformed into valuable commodities becomes residual product sent directly to a landfill. Constraints (9) limit the amount of each waste product delivered to a recovery centre given its capacity. Constraints (10) and (11) assure respectively that, a minimum percentage of the total amount of every disposal must be transformed into valuable commodities and that only a maximum quantity of each valuable commodity can be sold in the market. These constraints prevent that only the most valuable commodities are recycled and prevent also the sending of great amounts of disposal to landfills without recycling which is cheaper, in most of the cases. Constraints (13) guarantee that the maximum capacity of each collection centre for each type of disposal is not exceeded. Constraints (15) limit the number of modules for each technology that can be installed.

Constraints (16) guarantee that the residual disposal is sent from the recovery centres only to installed landfills. Constraints (17) impose that only an incinerator can be installed. Constraints (12) and (14) are consistency constraints.

4.6. Variable domains

$$y_i^C \in \{0,1\} \quad i \in \mathcal{I}_C$$

$$y_i^R \in \{0,1\} \quad i \in \mathcal{I}_R,$$

$$y_i^I \in \{0,1\} \quad i \in \mathcal{I}_I$$

$$y_i^L \in \{0,1\} \quad i \in \mathcal{I}_L$$

$$w_{ip}^C \in \{0,1\} \quad i \in \mathcal{I}_C,\ p \in \mathcal{P}_W$$

$$z_{ipg}^R \in \{0,1\} \quad i \in \mathcal{I}_R,\ p \in \mathcal{P}_W,\ g \in \mathcal{G}$$

$$n_{ipg}^R \in \mathbb{Z}^+ \quad i \in, \mathcal{I}_R,\ p \in \mathcal{P}_W,\ g \in \mathcal{G}$$

$$x_{ijps}^{OC} \geq 0 \ i \in \mathcal{I}_C, j \in \mathcal{J},\ p \in \mathcal{P}_W,\ s \in \mathcal{S}$$

$$x_{ijps}^{OR} \geq 0 \ i \in \mathcal{I}_R, j \in \mathcal{J},\ p \in \mathcal{P}_W,\ s \in \mathcal{S}$$

$$x_{ii'ps}^{CR} \geq 0 \ i \in \mathcal{I}_C, i' \in \mathcal{I}_R,\ p \in \mathcal{P}_W,\ s \in \mathcal{S}$$

$$x_{ii's}^{RI} \geq 0 \ i \in \mathcal{I}_R, i' \in \mathcal{I}_I,\ s \in \mathcal{S}$$

$$x_{ii's}^{RL} \geq 0 \ i \in \mathcal{I}_R, i' \in \mathcal{I}_L,\ s \in \mathcal{S}$$

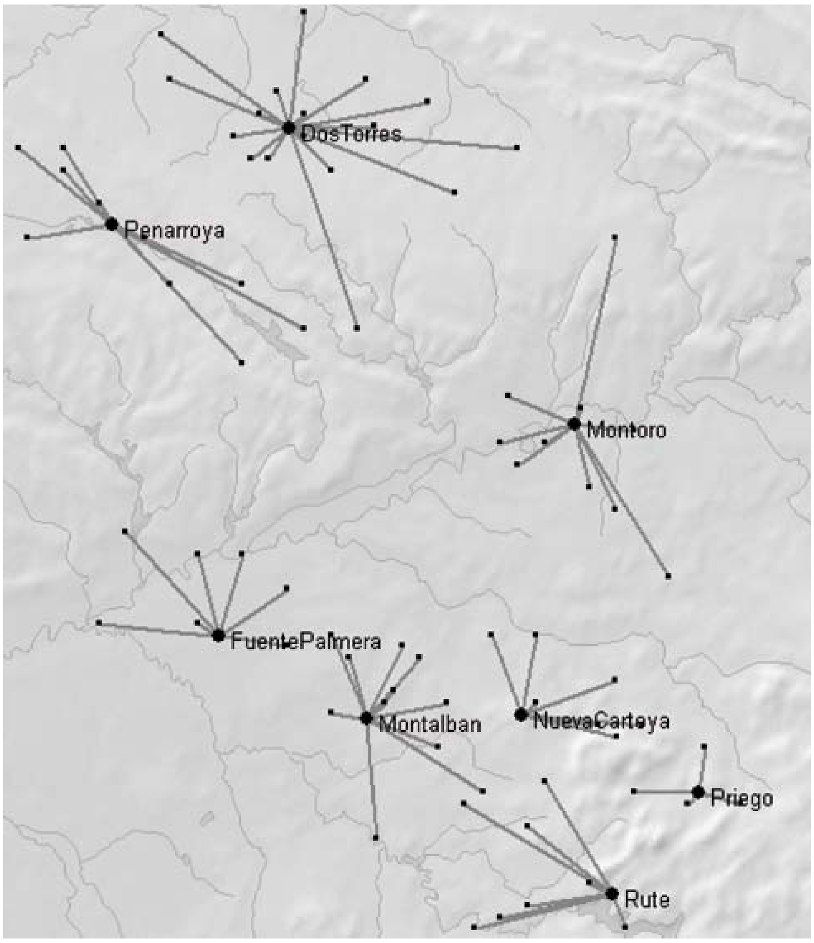
$$x_{ipqgs}^{RV} \geq 0 \ i \in \mathcal{I}_R, q \in \mathcal{P}_W,\ p \in \mathcal{P}_F,\ g \in \mathcal{G},\ s \in \mathcal{S}$$

5. Computational results

A case study has been used to test whether the problem can be addressed with this model. This model is based on data from the Spanish province of Córdoba, where a company is in charge of treating all sorts of waste produced in the region (but those from the capital), which means a total number of 78 towns.

In this case, the waste products were urban solid waste, plastics and similar products, glass, and paper. After being recovered, these products can be converted into five different valuable commodities, namely, sellable glass, compost, sellable paper, sellable plastic and similar products and, finally, products that are incinerated.

The model has been built in AIMMS 3.8 and solved using a standard solver, ILOG CPLEX 10.1 on an Intel Core 2 Duo 6320 1.86 GHz 2Gb RAM running under Windows XP.The computational times varied highly depending on the data and the objective function, ranging from 800 sec to several weeks. So far, the three objective functions have been studied separately. As an example, when obtaining the less costly solution, a single recovery centre is located in Montalban, where there also is a landfill. There are seven collection centres and no incinerator is installed.



6. Conclusions and further research

The model addressed in this paper is capable of helping make a wide range of decisions in Reverse Logistics related to what facilities to installed, where to locate them, with what technology and what capacities.

There are two major issues for future research. First, it would be of great interest obtaining non-dominated for the criteria considered. Second, since solving the model is very time-consuming for large instances, it should be studied how effectively perform another methods, such as Benders, L-shaped method, Particle Swap Optimization.

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